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Dealing with Topological Singularities in Volumetric Reconstruction

H. Lopes, L. G. Nonato, S. Pesco, and G. Tavares

Abstract. In this work we introduce a new representation for 3-dimensional stratified manifolds based on Morse theory. This representation, which we call *Handle-Strata*, includes a new data structure and a set of operators. Applications of this representation on the volumetric reconstruction from planar sections are presented.

§1. Introduction

Given a set of planar sections of an *object*, by definition a smooth 3-dimensional stratified manifold, the volumetric reconstruction problem consists in building a *geometric model* that is an approximation for this object. In this paper we work with piecewise-linear approximations.

There are several strategies for solving the 3-dimensional reconstruction problem, such as: heuristic, voxel, implicit, parametrical and optimal. Some of these techniques build the surfaces, which are the boundary of the solid object, while others generate a 3-dimensional cell decomposition of the object volume. Two of the main softwares in this area are the 1) Nuages software [8], developed by the PRISME project at INRIA Sophie Antipolis based on surface reconstruction and Volvis [11], and 2) software developed by the VolVis project at the Visualization Lab, Computer Science Department, SUNY at Stone Brook based on voxel reconstruction.

Three problems are intrinsic to the reconstruction process, namely: correspondence, tiling and branching. Correspondence consists in defining the connected components of the model. Tiling means to triangulate the strip between two adjacent slices with respect to some criteria. The branching problem is related to the identification of the object's *singularities.*

Boissonnat introduced an important heuristic technique based on computational geometry concepts of proximity [1]. This technique makes use of the 3-dimensional Delaunay Triangulation and Voronoi Diagram to generate

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the geometric model. For a definition of Voronoi Diagram and Delaunay Triangulation of a discrete set of different points in $\mathbb{R}^3$ see [3].

The advantages of using the Delaunay triangulation in reconstruction problems are: regions which are geometrically well positioned, with respect to some proximity measure can be found through topological tests; a volumetric triangulation connecting the regions is automatically generated; the volumetric triangulation is appropriated for applications in simulations.

However, without a suitable object representation, the advantages above cannot be fully realized. One of the main reasons is that the representation has to deal with the topological singularities that may appear during the process of reconstruction or even in post-processing applications, e.g., in applying finite element methods to deform the objects.

The main purpose of this work is to introduce a new representation (data structure and its operators) for the cell decomposition of an object. This representation is called *Handle-Strata* (HS-Rep for short). A second goal is to discuss the applications for this new representation in the reconstruction process.

The paper is organized as follows. Section 2 introduces the *Handle-Strata* representation. Section 3 discusses one reconstruction method based on Delaunay Triangulation, and identifies the role of singularities in the reconstruction process. Section 4 shows the applications of this new representation to volumetric reconstruction. Finally, in Section 5 we show images of some reconstructed objects.

§2. Handle-Strata Representation: Data Structure and Operators

In [2], Castelo, Lopes and Tavares introduced a representation for surfaces with boundary based on Morse theory [4]. Lopes and Tavares in [6] extended it to deal with 3-manifolds with boundary. In [9], Pesco devised a representation for stratified surfaces also on Morse theory.

The representation we introduce in this paper is for the 3-dimensional cellular decomposition of an object in $\mathbb{R}^3$. The HS-Rep is an extension of [9] to deal with stratified 3-manifolds. A 3-dimensional cellular decomposition of a subset $\mathcal{K}$ in $\mathbb{R}^3$ is a collection $\mathcal{C}$ of i-dimensional cells ($i = 0, 1, 2, 3$) in $\mathbb{R}^3$ under the following conditions:

1) $\mathcal{K} = \cup\{\sigma \in \mathcal{C}\}$,
2) If σ and $\tau \in \mathcal{C}$ then $\sigma \cap \tau \in \mathcal{C}$, where this intersection is either empty or a sub-cell of both σ and τ,
3) Any compact subset of $\mathcal{K}$ intersects only a finite number of cells.

A subset $\mathcal{M} \subset \mathbb{R}^3$ is said to be an n-dimensional combinatorial manifold with boundary (n=0,1,2,3) if it has an n-cell decomposition in which the neighborhood of each point is homeomorphic either to an n-sphere or to an n-semi-sphere. The 0,1,2 and 3 dimensional manifold will be called, respectively, point, curve, surface and volume. A combinatorial stratification of a set $\mathcal{K} \subset \mathrm{R}^3$ is a chosen finite collection of combinatorial submanifolds with boundary such that their union is $\mathcal{K}$ and the intersection of two of its elements

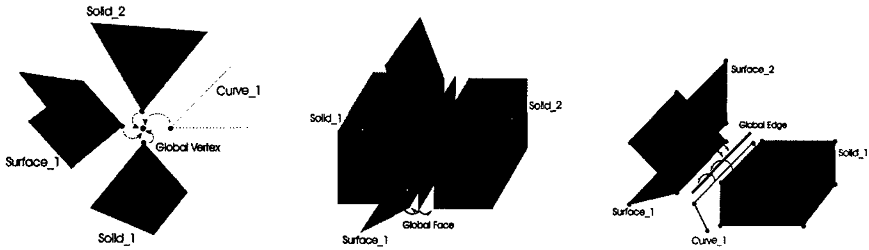

Fig. 1. Global Elements.

belongs to the cellular decomposition. Each manifold in this combinatorial stratification is called a stratum. A stratum could be a point, a curve with or without boundary, a surface with or without boundary or a 3-manifold with boundary.

In this paper, an object is defined as a set $\mathcal{O} \subset \mathrm{R}^3$ endowed with a 3-dimensional combinatorial stratification.

Now we describe the data structure behind the HS-Rep for the cell decomposition and the stratification of an object. The data structure nodes are classified in three types: strata, local cells and global cells.

- Strata nodes (Point, Curve, Surface and Volume) describe the manifold components of the object stratification.
- Local cell nodes represent the cells of a stratum. For instance, a curve has two types of local cell elements, the curve-vertex and the curve-edge. A surface has three types of local cell nodes: surface-vertex, surface-edge and surface-face. Also, there are three kinds of local elements for volumes: volume-vertex, volume-edge and volume-face.
- Global cell nodes (Global Vertex, Global Edge and Global Face) are used to represent the cellular decomposition of the object. Also, global cell are used to identify the local cells of different strata. A global cell is said to be singular if it has more than one local cell associated with it. Thus, on this data structure the singularities are explicitly represented.

In Figure 1 some examples of the use of the global vertex, global edge and global face cells are shown. In Figure 2 we have the hierarchy scheme of the data structure associated with the HS-Rep.

The main characteristic of this new data structure is the explicit representation of the object stratification. The stratification allows the representation of singular objects and manifolds of different dimensions in the same environment. Those manifolds are linked together through the global cells. One advantage of using objects as defined in this paper is that it keeps to a minimum the redundant information stored in each cell.

The representations introduced by Weiler [12] and Gursoz [5] also deal with singular objects (non-manifolds), but they don't identify manifold components.

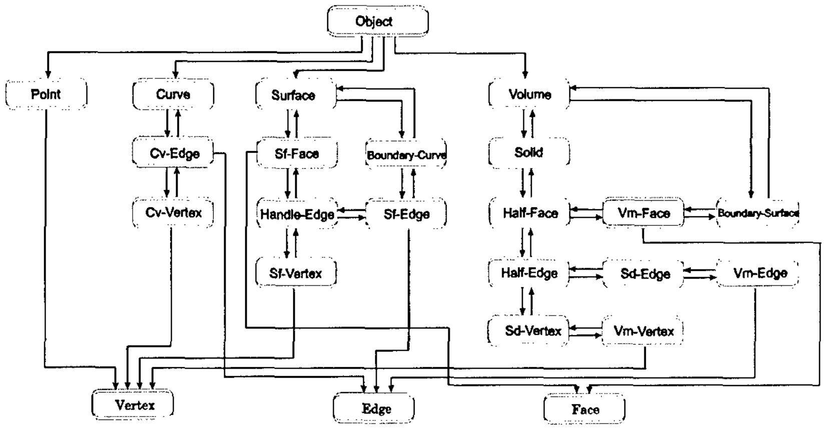

Fig. 2. HS-Rep Data Structure.

A set of operators to build and unbuild an object on this representation, called Morse operators, will now be described. These operators are validated by the Piecewise-Linear Handlebody Theory [10] and they correspond to gluing handles on manifolds with boundary. Morse operators are divided in two groups: local and global operators. Local operators build and unbuild strata. Global operators perform the union of strata.

The local building operators are used to identify two boundary m-cells ($m = 0, 1, 2$) of a respective regular $(m + 1)$-dimensional manifold. The local building operators for curves create an interior vertex by the identification of two boundary vertices, which can be both on the same curve component, or on different connected components.

There are five situations where two boundary edges of surfaces can be identified. For each one a local building operator is defined. These five cases are distinguished by the following criteria: 1) the two boundary edges don't have vertices in common but they are on different surface components; 2) the two boundary edges don't have vertices in common but they are on different boundary curve components of a surface (on this situation, a genus is created on a surface); 3) the two boundary edges don't have vertices in common but they are on the same boundary curve; 4) the two boundary edges have only one vertex in common and, finally, 5) the two boundary edges have two vertices in common. More details on those operators on surfaces can be found in [2].

For 3-manifolds, there are also five situations where two boundary faces can be identified. Each case defines a local building operator for 3-manifolds. These cases are distinguished according to the following criteria: 1) the two boundary faces are on the same connected component of the manifold; 2) the two boundary faces are on the same boundary surface component; the two boundary faces have or have not edges in common. A detailed discussion of these operators for 3-Manifolds can be found in [6].

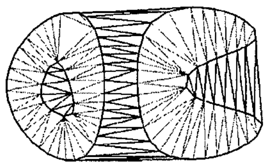

Fig. 3. Edges of $\mathcal{T}$ on a planar section.

The global operators identify m-dimensional global cells ($m = 0, 1, 2$). Those operators make the union of different strata through the use of the global elements on the data structure.

§3. Volumetric Reconstruction from Planar Sections

In this section a heuristic based on the Delaunay Triangulation for the volumetric reconstruction is discussed. This heuristic was introduced in [7] and is now rediscussed in terms of the representation introduced in this paper.

Here the reconstruction process will be restricted to two consecutive planar sections. The object is built from contours in the planar sections by applying the appropriate heuristic to the Delaunay triangulation. These contours are simple polygons that bounds the planar regions to be connected, and can be oriented coherently.

The first phase of the reconstruction process generates a 3-dimensional Delaunay triangulation that contains all edges of the contours on two consecutive slices. This triangulation will be called the restricted Delaunay triangulation of the slices, and will be denoted by $\mathcal{T}$. To obtain such a triangulation, the following algorithm has been devised:

1) Build a 3-dimensional Delaunay triangulation $\mathcal{D}$ using the vertices of all contours,
2) Mark the edges of the contours that are not contained on $\mathcal{D}$,
3) Subdivide all marked edges, inserting new vertices on the contours,
4) Make local modifications on $\mathcal{D}$ to obtain a new Delaunay triangulation that includes those new vertices,
5) Repeat these steps until the triangulation contains all contour edges.

Boissonnat [1] shows that the missing edge subdivision strategy, used in the above algorithm, obtains a Delaunay Triangulation that includes all contour edges.

The second phase of the reconstruction process classifies the edges of $\mathcal{T}$ contained in the planar sections as internal, external or contour edges according whether they are internal, external or on the contours. Figure 3 shows a set of contours and the external, internal and contour edges of $\mathcal{T}$ that are on a planar section.

To generate a model, which satisfy the resampling condition, i.e. whose intersection with the given planes corresponds exactly to the same given contours, it is necessary to identify the connected components and modify the

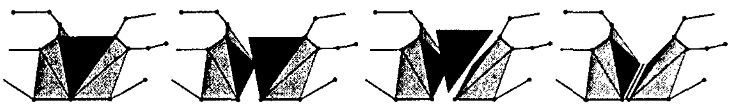

Fig. 4. Singular edge coming from a reverse tetrahedron elimination.

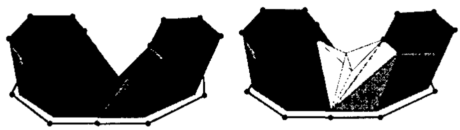

Fig. 5. Tetrahedron subdivision avoiding singular edge creation.

triangulation. For this we will introduce the concept of reverse tetrahedra and geometrically well positioned contours.

A tetrahedron of the triangulation $\mathcal{T}$ is called a reverse tetrahedron if they have edges on different slices which are not contour edges. Two contours on consecutive slices are said to be geometrically well positioned if they are connected by a reverse tetrahedron in $\mathcal{T}$.

Intuitively, it is appropriate to maintain on the same connected component contours based on distinct slices which are geometrically well positioned. In the heuristic introduced in [7], reverse tetrahedra play an essential role on the 3-manifold components definition because they identify when two contours are connected to each other.

A singular edge on $\mathcal{T}$ is defined as an edge whose associated link is not homeomorphic either to a sphere or to a semi-sphere on the corresponding 3-dimensional manifold.

Next we can use the representation introduced above to deal with the branching problem. We propose a heuristic using singular edges which at the end generates a triangulated manifold between the slices:

1) Remove all tetrahedra with at least one external edge. The removal of one tetrahedron may generate a singular edge, see Figure 4.

2) Identify singular edges.

   a. If the singular edge is interior to the contour, reinsert the corresponding reverse tetrahedron, subdivide its external edge and push the new vertex to a position in between the slices, see Figure 5. The role of this translation is to guarantee the resampling condition.

   b. If the singular edge is on a contour, split the connected components as in Figure 6.

Finally, the whole object is reconstructed by putting together the objects built between consecutive slices.

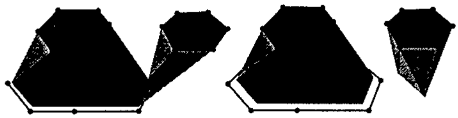

Fig. 6. Avoiding edge singularity on a contour edge.

§4. HS-Rep Applications to Volumetric Reconstruction

The algorithm of Nonato and Tavares [7] has no explicit data structure dealing with edge singularities. Thus, the main contribution of this section is to present several instances where the HS-Rep representation simplifies the reconstruction process.

Initially, all contours are created on the data structure using the building operators for curves. The vertices of these contours are used to build the initial 3-dimensional Delaunay triangulation $\mathcal{D}$. After that, the contour vertices are identified with the vertices of $\mathcal{D}$ through the global vertex operator.

To verify that a contour edge is on the triangulation $\mathcal{D}$, one has to look at the star of each global vertex and check for incidence to verify if this contour edge is on the boundary surface of the volume. If the contour edge is on the boundary surface then it is associated with the corresponding contour edge on the slice curve by using a global-edge building operator. Otherwise, the contour edge must be subdivided and the Delaunay triangulation will be locally modified to include this new point. This process will continue until the triangulation $\mathcal{T}$, which contains all contour edges, is obtained. The non-contour edges whose vertices are on the same slice can be classified either as internal or external traversing the list of edges of the boundary surface of $\mathcal{T}$, which is then explicitly represented on the data structure.

Section 3 points out that the first step in the identification of the connected components is the elimination of the tetrahedra with at least one external edge. To remove a tetrahedron, split its internal faces into boundary faces using the local Morse operators for 3-manifolds. To reconstruct it as a manifold, singular edges have to be detected. Global singular edges are detected by performing a counting on the number of incident 3-manifold strata.

Now local building operators for 3-manifolds are used to insert tetrahedra and subdivide its edges. The new vertices added in this subdivision are translated to a position inbetween the slices. When the singularity is a contour edge, a global operator is used to split the manifold components.

Finally, the objects obtained on consecutive slices are glued together. The process of gluing those objects consists in applying local building operators to all boundary faces on the contour interior.

The Handle-Strata computational environment is suitable for dealing with either the strategy given by Nuages [8] or Nonato and Tavares [7]. Moreover, this representation is well suited for4 integrating different techniques under the same common topological kernel. Issues like graphics interface, visualization, objects physical properties, deformations, and so on, can now be addressed as

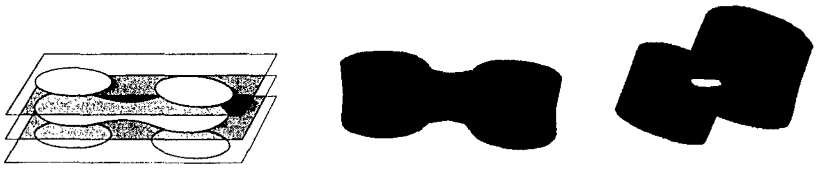

Fig. 7. Reconstruction of the bitorus' slices using Nonato and Tavares heuristic.

Fig. 8. Reconstruction of bitorus' slices using Nuages.

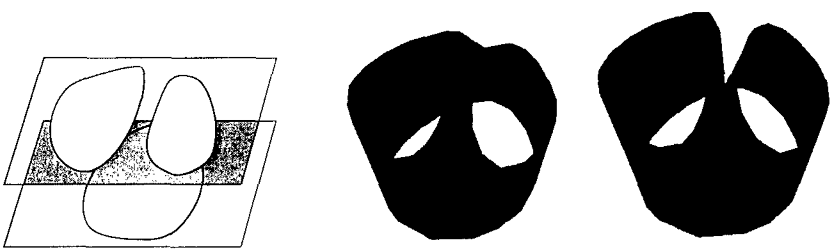

Fig. 9. Reconstruction using, respectively, Handle-Strata and Nuages.

attributes or applications of the Handle-Strata representation.

The examples below come from three slices of a bitorus, in which the bottom and the top slice have two curves and the intermediate slice has only one curve. For these images, only the boundary faces not on the slices are visualized. In Figures 7 and 8 we show the input slices and two views of the models reconstructed using, respectively, the proposed algorithm and that of Nuages.

Nuage's reconstruction inserts edge singularities at an intermediate level. The reconstruction using *Handle-Strata* avoided that singularity through tetrahedra insertion and subdivision.

The second example take two slices of a torus, in which the one on the top has two contours that are geometrically well positioned with the unique contour on the bottom slice. The three pictures of Figure 9 show, the slices, our reconstructed model, and Nuage's result, respectively.

The intersection of the bottom plane with the model created with that of Nuages is not the original curve, i.e, it does not satisfy the resampling condition. The new heuristic creates a saddle in order to avoid this singularity.

Fig. 10. Spine Vertebra Reconstruction.

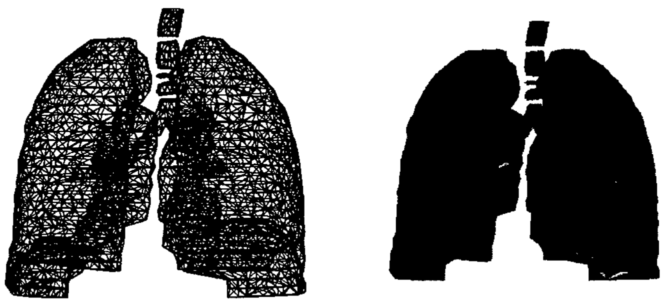

Fig. 11. Lung Reconstruction.

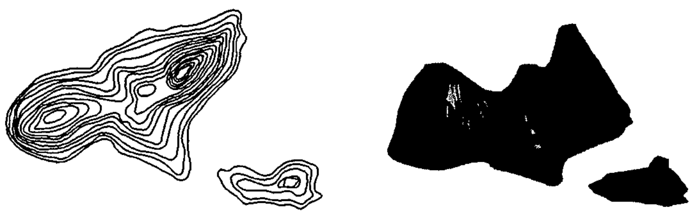

Fig. 12. Sugar Loaf Reconstruction.

The execution time and the number of tetrahedra in the final objects for both algorithms are essentially the same.

§6. Examples

Figure 10 shows the reconstruction of a Spine Vertebra. Figure 11 shows the reconstruction of a lung. Figure 12 shows an example of a terrain reconstruction given by its contour levels.

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H. Lopes, L. Nonato, S. Pesco and G. Tavares
Pontifícia Universidade Católica do Rio de Janeiro
Departamento de Matemática
Rua Marquês de São Vicente, 225, Gávea
Rio de Janeiro, RJ, Brazil, CEP:22.453-900
`{lopes,nonato,pesco,geovan}@mat.puc-rio.br`